Imitation and Efficient Contagion

Tristan Boyer and Nicolas Jonard

Ipag, Université du Luxembourg

9 June 2010

Imitation and Efficient Contagion

Tristan Boyer[a], Nicolas Jonard[b,1]

[a] *Institut de Préparation à l'Administration et à la Gestion, 184, boulevard Saint Germain, 75006 Paris*
[b] *Université du Luxembourg, 162a, avenue de la Faïencerie, L-1511 Luxembourg*

Abstract

In this paper we study the conditions under which efficient behavior can spread from a finite initial seed group to an infinite population living on a network. We formulate conditions on payoffs and network structure under which overall contagion occurs in arbitrary regular networks. Central in this process is the communication pattern among players who are confronted with the same decision, i.e. who are at the same distance from the initial seed group. The extent to which these agents interact among themselves (rather than with players who already have faced or subsequently will face the decision problem) is critical in the Prisoner's Dilemma. In the Coordination Game the key element is the cohesion of the efficient cluster, a property which is different from the one identified in the Prisoner's Dilemma. Additional results are obtained when we distinguish the interaction and information neighborhoods. Specifically, we find that contagion tends to be favored by fast neighborhood growth if an assumption of conservative behavior is made. We discuss our findings in relation to the notions of clustering, transitivity and cohesion.

Keywords: imitation, contagion, regular graphs, local interaction game
JEL: C72, C73, D70

1. Introduction

Casual observation suggests that most economic and social interactions take place locally, within restricted subsets of a larger population. It is convenient to think of the set of pairwise relationships among agents as a network. An agent's neighborhood might include family and friends, colleagues, business partners, geographic neighbors, etc. Within his neighborhood, an agent shares, exchanges and develops information, knowledge and other resources, new behaviors are learned and strategic interactions take place.[2] Improved corporate governance practices spread across the economic system via board interlocks; increased awareness of Society towards environmental issues was achieved through associations, communities and friendship ties.

When it comes to the adoption of new behavior or technology, interaction being confined to neighborhoods implies that diffusion takes place gradually. Starting from a seed group, diffusion reaches the group's neighborhood, the neighborhood's neighborhood, and so on. The sequential process through which iterated neighborhoods convert resembles an epidemics, progressing through the population as contacts occur between infected and susceptible individuals. The extent and speed of contagion therefore depend on the properties of the iterated neighborhoods of the initial seed group.

In this paper, we formulate results on imitative contagion by efficient play in arbitrary regular networks,[3] based solely on the structure of the iterated neighborhoods of some initial, finite group of efficient players. We focus on one-shot two-by-two symmetric games. Efficient play implies the choice of cooperation in the Prisoner's Dilemma, and the choice of the action associated with the Pareto-dominant equilibrium in the Coordination Game. The decision to adopt one particular behavior depends on the payoff advantage of that behavior over alternative courses of action. We assume imitative learning: agents adopt successful actions without necessarily understanding why they are successful, i.e. imitation rather than best reply drives learning. We identify structural properties of iterated neighborhoods which are key to contagion by efficient behavior. In the Prisoner's Dilemma, efficient behavior spreads if and only if it has a chance of being imitated, which requires that it generates high payoffs. This can only happen if players are, to a certain extent, segregated, and remain so as contagion unfolds. This in turn requires that agents play mostly with agents who are at the same distance from the initial seed group.

Email address: nicolas.jonard@uni.lu (Nicolas Jonard)
[1] Financial support from the Fonds National de la Recherche (FNR) under Project BaBe C08/IS/13 is gratefully acknowledged.
[2] Different neighborhoods could serve different purposes, for instance when interaction and information neighborhoods do not coincide. We come back to this later.
[3] Networks in which all nodes have the same degree, i.e. the same neighborhood size.

Put another way, an efficient outcome can be reached when those facing the challenge interact among themselves rather than with those who faced it yesterday or those who will face it tomorrow. In the Coordination Game, efficient contagion is possible if the cluster of efficient players has few outward connections, a condition easier to fulfill.

This paper is related to a large set of references, which we (selectively) review below, focusing on how imitation and local interaction have been incorporated so far in the literature.

In the Prisoner's Dilemma, best reply precludes cooperation regardless of how agents are matched (be it locally or globally). Best reply however assumes that a great deal is known about the structure of the game. For many applications this is neither reasonable nor necessary. Economic agents are often unfamiliar with the set of actions they can choose from, and with the details of the strategic environment they are immersed into (see Simon's 1955 seminal paper). When information is incomplete but the performance and action of others are (perhaps indirectly) observable, imitation is a reasonable behavior: it saves time and attention, and simplifies decision making. Mimetism in financial markets, best practices identification and benchmarking in industry are typical illustrations of imitative behavior (on the latter, see Lieberman and Asaba 2006). Experimental evidence supports the idea that agents use imitation in decision making (see Pingle and Day 1996). There is also evidence that, within a large class of possible imitation rules, agents follow the principle of "imitating the best" (Apesteguia et al. 2007), a principle which is showed to be the only "non-exploitable" standard learning algorithm (Duersch et al. 2010).[4]

Imitation alone is however not enough. To protect efficient play from too much exposure to the other action (which would implied lowered payoffs), interaction also needs to be local. This can be achieved with a fixed structure like a network, or in a more fluid way through random pair (or small group) formation. An early analysis of imitation in conjunction with local interaction is in Nowak and May (1992, 1993). Using numerical simulation, it is showed that for a substantial part of the possible payoff values (the magnitude of the defection premium plays a significant role here) and most initial conditions, defectors and cooperators can coexist forever on a lattice, either in static irregular patterns or in dynamic patterns with chaotic or cyclical fluctuations around predictable long-term averages. Similar results obtain in both synchronous and asynchronous environments (Nowak et al. 1994; Hubermann and Glance 1993), when various changes are introduced in the learning procedure (noise, memory, etc.), and on a variety of network structures. Variations on these elements have been explored in numerous simulation studies, of which recent examples are Abramson and Kuperman (2001), Ohtsuki et al. (2006) and Jun and Sethi (2007). Roca et al. (2009) perform a very systematic simulation study which explores the various degrees of freedom of the problem, and emphasize the central role played by clustering (or transitivity) in sustaining efficient play.

Taking an analytical approach, Eshel et al. (1998) examine the survival of cooperation in a Prisoner's Dilemma played on the circle and show that at least two thirds of cooperators exist in any stochastically stable configuration. The authors however are constrained by their methodology to stick to the circle (the one-dimensional periodic lattice) with two or four nearest neighbors. Outkin (2003) characterizes the necessary and sufficient conditions for the existence of a mixed configuration on the circle with 4 neighbors, showing how the size of the equilibrium cluster of cooperators relates negatively to the defection premium. Eshel et al. (1999) consider larger neighborhoods on the line and distinguish interaction from information. They make however an assumption of "conservatism" according to which only players at the border of clusters can change strategy. Depending on the relative size of the information and interaction neighborhoods, risk-dominance and Pareto-dominance can prevail in Coordination Games, while cooperation emerges in the Prisoner's Dilemma provided learning takes place at long enough distances. Mengel (2009) also distinguishes interaction and information in an imitation-driven approach with players located on the circle and a few additional structures. Whenever agents use information beyond their interaction neighbors, the unique stable outcome is inefficient. Introducing sufficient conformism (which in effect implies that the payoffs are distorted, and not those of a Prisoner's Dilemma anymore as they include a bias towards the majority behavior) is a way of sustaining efficient play.

Similar issues arise in coordination games, where best reply also leads to inefficiency (i.e. the selection of the risk-dominant equilibrium). Noisy best reply in relation to equilibrium selection is studied in global contexts by Kandori et al. (1993), Young (1993) and Binmore and Samuelson (1997) (see Bergin and Lipman 1996 on the very particular role played by noise in these models). Myopic best reply in local contexts has been examined in the early papers of Blume (1993, 1995); Anderlini and Ianni (1995); Berninghaus and Schwalbe (1996); Young (1998); Ellison (1993, 2000); Morris (2000). All conclude that the risk-dominant equilibrium is selected. Most of these papers consider lattices, but there are two noticeable exceptions. Morris (2000) formulates very general results on contagion in arbitrary, infinite-order networks. The approach uses the notion of cohesion, i.e. the extent to which groups of individual have interactions within the group rather than outside the

[4]See also Schlag (1998) for a motivated and motivating discussion of imitation, and Samuelson's (1997) book. The recent study by Bergin and Bernhardt (2009) shows that imitation tends to favor cooperative solutions, and that this tendency is stronger when there is a longer memory of past interactions.

group (see also Young 1998). The paper shows that a necessary and sufficient condition for contagion by the q-dominant action ($q < 1/2$) starting for a finite group X, is that the complement of X (where the inefficient, $(1-q)$-dominant action is played) contains no group which is at least $(1-q)$-cohesive. Uniformity of interaction and low neighbor growth are showed to support contagion by efficient play. Durieu et al. (2005) also provide general results, using the graph-theoretic concept of dominating set to provide a complete characterization of step-by-step contagion in finite networks, and pointing at the possibility of non-homogeneous equilibria as well as two-period cycles. Recent contributions include López-Pintado (2006), who uses a mean-field approach to show that in random networks of arbitrary degree distribution there is a threshold for the degree of risk-dominance q below which contagion happens (the threshold being dependent on the degree distribution). Cartwright (2007) explores the relationship between the waiting time for a transition from the efficient to the risk-dominant equilibrium, and network structure, finding that small world types of architectures yield fastest transitions.

When learning is driven by imitation, the opposite conclusions obtain. Robson and Vega-Redondo (1996) have a very parsimonious model where random pairs of agents play a Coordination Game and publicize the outcome of the interaction. This gives the possibility that efficient players meet, collect the largest payoff and are imitated by others. Although there is no fixed network over which interactions take place, interactions are local, and efficient play is sustained. Alós-Ferrer and Weidenholzer (2008) distinguish the interaction and the information neighborhoods, extending the latter strictly beyond the former, and show that efficient play spreads contagiously to the entire population when interaction is not "too global", i.e. neighborhoods are small relative to the maximal number of disjoint neighborhoods, and information is fluid enough.

In this paper we analyze arbitrary regular networks of infinite order. Focusing on the case of a constant premium to defection, we are able to fully characterize the process through which the iterated neighborhoods of a given finite-sized initial set of efficient players convert, propagating the epidemics through the entire network. In the Prisoner's Dilemma, the results emphasize the importance of a property of iterated neighborhoods which is related to, though distinct, from cohesion. For the Coordination Game, neighborhood growth plays a central role. When the interaction and information neighborhoods are distinguished, neighborhood growth plays an important part in the Prisoner's Dilemma as well.

2. The model

2.1. Network definitions

Consider a countably infinite set of players S and a set g of ties (links) between unordered pairs in S. The pair (S, g) defines a *network*. Writing ij to represent the tie between i and j, $ij \in g$ indicates that i and j are linked in the network. The *neighborhood* of i consists of the players to whom i is directly connected, denoted $N_i = \{j \neq i : ij \in g\}$. The size of the neighborhood of i is the number of ties held by i, also called its *degree*, and is denoted $n_i = \#N_i$ (in the following, lower case letters are used for set sizes). In the paper, degree is assumed to be identical across S, i.e. $n_i = n, \forall i \in S$: the network is n-regular. The neighborhood of a set of players X is the union of neighborhoods of the players in X, denoted N_X.

A *path* in g is a sequence of links $i_1i_2, \cdots, i_{p-1}i_p$ between unordered pairs, without cycles, such that $i_li_{l+1} \in g, \forall k = 1, \ldots, p-1$. Denote $d_{i,j}$ the number of links in the shortest path between i and j, $d_{i,j}$ is the (geodesic) *distance* between i and j in the network (S, g). It is straightforward to see that $d_{i,j} = 1 \, \forall j \in N_i$. The convention is to set $d_{i,j} = \infty$ whenever there is no path connecting i and j. We suppose that $d_{i,j} < \infty, \forall i, j \in S$, i.e. the network is *connected*. The distance between a finite set $X \subset S$ of players and the individual player $i \notin X$ is $d_{X,i} = \min_{j \in X}\{d_{i,j}\}$. We write N_X^d for the d-iterated neighborhood of X, the set of players who are exactly at distance d of the player(s) in X they are closest to, $N_X^d = \{i \in S : d_{X,i} = d\}$. We define the *d-shell* of X as the set of players who are *within* distance d of X, *augmented* with X,

$$X_d = X \bigcup_{1 \leqslant l \leqslant d} N_X^l. \tag{1}$$

For a given finite set X and any i in the d-shell of X (thus $d_{X,i} = d$), we define the *in-degree* n_i^{in} of i as $\#N_i \cap N_X^{d-1}$, the *out-degree* n_i^{out} of i as $\#N_i \cap N_X^{d+1}$, and the *intra-degree* of i as $n_i^* = n - n_i^{in} - n_i^{out} = \#N_i \cap N_X^d$. This allows to distinguish the interactions of i within the iterated neighborhood it belongs to, from the interactions of i with the previous and the following iterated neighborhoods. We assume that each player has a link to both the next and the previous iterated neighborhoods, i.e. $n_i^{out}, n_i^{in} \geqslant 1, \forall i \in S$. As a consequence, the d-shell of X is bounded, its size satisfying $x + d \leqslant x_d \leqslant x\left((n-1)^{d+1} - 1\right)/(n-2)$.

Finally, it is useful to introduce the notions of clustering coefficient, transitivity and cohesion in a network. The clustering of i's neighborhood, cl_i, is the number of neighborhood ties which exist among i's neighbors, $\#\{jk \in g : j, k \in N_i\}$, divided by the number which could in principle exist, $n(n-1)/2$. Put another way, cl_i is the number of (unordered) triangles connected to player i divided by the number of (unordered) open triangles centered on player i. The clustering coefficient in a network is the average of that statistic taken over all players. An alternative measure which is sometimes more easily computed is transitivity. Transitivity is the fraction of transitive triples in the whole network, i.e. 3 times the number of (unordered) triangles in the network, divided by the total number of (unordered) open triangles centered on any player (the factor 3 appears because each triangle involves 3 open triangles). Last, a group of players in a network is x-cohesive (where x is between 0 and 1) if all players have at least a percentage x of their interactions within the group. These three notions, while yielding different values in general, revolve around the issue of local neighborhood growth: the number of agents reached in d steps tends to grow only slowly when networks are clustered, transitive, or contain many cohesive groups.

2.2. The game and learning

Players repeatedly engage in the symmetric two-by-two Prisoner's Dilemma depicted in Table 2.2, with payoffs satisfying $\tau > \pi + \beta$ and $\pi, \beta > 0$. D is a dominant strategy, while C is the only symmetric and efficient outcome (it dominates any symmetric mix of play). Assuming an identical defection premium (β) for any action of the column player, payoff differences can be compared in terms of the number of C-players involved.

	C	D
C	$\tau - \beta$	$\pi - \beta$
D	τ	π

Table 1: Payoffs to the row player in the Prisoner's Dilemma.

A (pure) *configuration* is a function $\omega : S \to \{C, D\}$. The *state* or *action* of player $i \in S$ is $\omega(i) \equiv \omega_i \in \{C, D\}$. Write $p(\omega_i, \omega_j)$ for the payoff of player i from a particular interaction with player j. The *aggregate payoff* to player i is

$$P_i = \sum_{j \in N_i} p(\omega_i, \omega_j) = \gamma_i (\tau - \pi) + n\pi - n\beta \cdot \mathbf{1}_{\{\omega_i = C\}} \tag{2}$$

where $\gamma_i = \#\{j \in N_i : \omega_j = C\}$ is the number of C-players in the neighborhood of i, and $\mathbf{1}_{\{condition\}} = 1$ if $\{condition\}$ is true, and 0 otherwise.

At the end of each period, each agent decides on a course of action for the next period. If agents were myopic expected utility maximizers, then the strictly dominant strategy D would be chosen systematically. Instead, the rule they follow is imitation. Players imitate the actions of others whom they observe to be earning high(er) payoffs. Specifically, in each period, each agent observes his own payoff, and the payoff and action of each every other agent in his neighborhood (though an agent does not play with himself, he also considers his own payoff and state when imitating). Agent i sticks or switches to C whenever the highest payoff in his augmented neighborhood $N_i^+ = \{i\} \cup N_i$ is achieved by a C-player. In case of a tie, i.e. if the highest payoff is achieved simultaneously by players of C and D, we assume C is chosen. So there is always a unique imitative reaction for any player i to any configuration ω.

Identify now a configuration with the group of players who choose C in that configuration. So configuration ω is identified with the group $Z = \{i : \omega_i = C\}$, and group Z is identified with the configuration ω where $\omega_i = C$ if $i \in Z$, and D otherwise. For any i, let $c(i)$ denote a C-player who collects the largest payoff among C-players in the augmented neighborhood of i. Formally, $c(i) \in \arg\max_{j \in N_i^+ \cap Z} P_j$. Symmetrically, let $d(i) \in \arg\max_{k \in N_i^+ \cap \bar{Z}} P_k$ be a D-player with largest payoff among the D-players. One of the two sets is necessarily non empty. Then the imitative reaction for player i to group Z is uniquely defined as

$$r(Z, i) = C \text{ if } P_{d(i)} \leqslant P_{c(i)} \tag{3}$$

and $r(Z, i) = D$ otherwise. Observe that the inequality $P_{d(i)} \leqslant P_{c(i)}$ can be rewritten as

$$\beta \leqslant \frac{\left(\gamma_{c(i)} - \gamma_{d(i)}\right)(\tau - \pi)}{n} \tag{4}$$

and let $\beta_\ell = \ell (\tau - \pi)/n$, for $\ell = 1, \ldots, n-1$ and $\beta_0 = 0$. When the defection premium $\beta \in (0, \beta_1]$, it is sufficient to have $\gamma_{c(i)} = \gamma_{d(i)} + 1$ to ensure that $P_{d(i)} \leqslant P_{c(i)}$: this is the most "favorable context" for collaboration. More generally, when the defection premium β lies in $(\beta_{\ell-1}, \beta_\ell]$, $P_{d(i)} \leqslant P_{c(i)}$ is equivalent to $\gamma_{d(i)} + \ell \leqslant \gamma_{c(i)}$. The larger the defection premium β is, the larger the number of extra C-players necessary for C to be more rewarding than D. For $\beta \in (\beta_{\ell-1}, \beta_\ell]$, we can thus formulate the imitative reaction correspondence as

$$R(\Omega) = \{i \in Z \cup N_Z : \gamma_{d(i)} + \ell \leqslant \gamma_{c(i)}\}. \tag{5}$$

3. Contagion

The question we seek to answer is whether a (connected) finite group of C-players can start an epidemic which, under the deterministic imitation dynamics described above in Equation 5, will lead to cooperation being played everywhere in the population. Central in answering this question is the local structure of the network, as cooperation can only propagate through neighborhood relationships. Morris (2000) addresses a similar question in the case of Coordination Games played under myopic best reply, whereas the present paper is concerned with imitation in both the Prisoner's Dilemma and the Coordination Game.

We will say that a connected finite-sized set X of players seeds shell-wise contagion when $R(X_d) = X_{d+1}$, for all $d \geqslant 0$. In this form of contagion, iterated neighborhoods at any distance convert homogeneously to C, one at a time, and never revert to D. Then we will say that there is shell-wise contagion in a given network (S, g) if there exists a finite set X such that X seeds shell-wise contagion.[5] An additional assumption we place is that players in the initial seed group X obtain the largest possible reward to cooperation: any $i \in X$ either is or has a neighboring C-player only interacting with C-players, and that this is enough for them not to switch to D.

3.1. Shell-wise contagion

We are now ready to formulate the central result of the paper, in which shell-wise contagion is characterized only in terms of the network structure. It is worth emphasizing that, although the formulation is very simple, the result is quite general on the effect of network structure on the possibility of successful diffusion of cooperation.

Proposition 1. *Suppose $\beta \in (\beta_{\ell-1}, \beta_\ell]$, for $\ell \geqslant 1$. There is shell-wise contagion in (S, g) if and only if there exists a connected finite-sized set $X \subset S$ such that*

$$\max_{k \in N_i^+ \cap N_X^{d_{X,i}}} \{n_k^{in}\} + \ell \leqslant \max_{j \in N_i^+ \cap N_X^{d_{X,i}-1}} \{n_j^{in} + n_j^*\} \tag{6}$$

for all $i \in S - X$.

Proof. Suppose contagion has reached the d-neighborhood of X ($\omega_i = C$ for all $i \in X_d$), where d is a non-negative integer number. For any $i_2 \in N_X^{d+1}$ (and thus playing D), i_2 switches from D to C if and only if Condition 6, $\max_{k \in N_{i_2}^+ \cap N_X^{d+1}} \{n_k^{in}\} + \ell \leqslant \max_{j \in N_{i_2}^+ \cap N_X^d} \{n_j^{in} + n_j^*\}$, holds. A trivial consequence is that then $n_k^{in} + \ell \leqslant n - 1$, for all $k \in N_X^{d+1}$. Turn now to any $i_1 \in N_X^d$ (i_1 thus plays C). Observe that i_1 sees a player in N_X^{d-1} who interacts only with Cs (if $d = 0$ and thus we are considering $N_X^0 = X$ and N_X^1, remember we have assumed that any $i \in X$ is or sees a C-player only interacting with C-players). Thus sticking to C is the best response to i_1 if and only if $\max_{k \in N_{i_1}^+ \cap N_X^{d+1}} \{n_k^{in}\} + \ell \leqslant n$. But we have just seen that $n_k^{in} + \ell \leqslant n - 1, \forall k \in N_X^{d+1}$. So the stability of Cs in N_X^d is implied by the conversion of Ds in N_X^{d+1}. This is true for any value of $d \geqslant 0$. Starting from a finite C-cluster X, Condition 6 triggers the conversion of the D-players in N_X^1 and guarantees the stability of the C-players in X. Remember Ds further away see no C-player and thus cannot switch. So $X_1 = R(X)$. In the next period, Condition 6 triggers the conversion of the D-players in N_X^2 and guarantees the stability of the C-players in X_1, thus $X_2 = R(X_1)$. It follows that $X_{d+1} = R(X_d)$ for all $d \geqslant 0$. Put another way, X seeds shell-wise contagion if and only if Condition 6 holds for all $i \in S - X$. $\square$

This simple and general result highlights the importance of intra-neighborhood interaction for the diffusion of cooperation. To illustrate, consider player i in Figure 1, where $n = 5$. Suppose the current C-cluster is $X_{d_{X,i}-1}$, so that the iterated neighborhood to which i belongs, $N_X^{d_{X,i}}$, is the frontier between the C- and D-playing regions. With the assumption that $\ell = 1$ the payoff difference between cooperation and defection is large enough to trigger a change as soon as there is at least one extra C-player. It is intuitive that players in $N_X^{d_{X,i}-1}$ generate high payoffs when they play mostly among themselves, i.e. have low out-degree, and thus have large in- and/or intra-degree. Here we see players in $N_X^{d_{X,i}-1}$ with out-degrees 1 and 2, and thus summed in- and intra-degree of 3 and 4. Similarly, players in $N_X^{d_{X,i}}$ generate low payoffs when they play among themselves, i.e. have low in-degree, and thus have large out- and/or intra-degree. Here we see players in $N_X^{d_{X,i}}$ with in-degrees of 1 and 2. Any C in $N_X^{d_{X,i}-1}$ plays at least with 3 Cs, whereas any D in $N_X^{d_{X,i}}$ plays at most with 2 Cs. Thus C can spread from $N_X^{d_{X,i}-1}$ to $N_X^{d_{X,i}}$, as happens in Figure 1.

[5] Another, less demanding form of contagion is the continuing growth of the initial C cluster who is joined by at least one player in each period. Unlike shell-wise contagion, which ensures conversion of the entire population, such a form of contagion would only ensure that an infinite-sized cluster forms, possibly coexisting with an infinite-sized D-cluster. The present paper focuses on shell-wise contagion, acknowledging that conditions for the other (weaker) form of contagion are much more difficult to characterize.

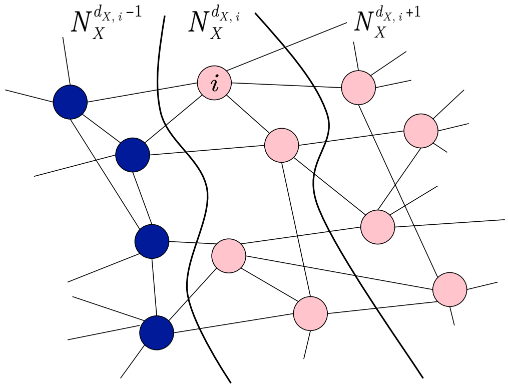

Figure 1: An illustration of shell-wise contagion when $\ell = 1$.

Interestingly, note that such interactions do not imply clustering in the iterated-neighborhoods. Though neighbor commonality is possible, and more likely for larger values of ℓ, it is not systematically implied by large intra-degrees. We now turn to a more detailed analysis of the importance of intra-degree for the diffusion of efficient play.

3.2. Partial characterizations

3.2.1. Homogeneous and partly homogeneous networks

The strongest homogeneity case is the one in which, for given X, the in-, out- and intra-degree are identical across all players: $n_i^{in} = n^{in}$, $n_i^* = n^*$ (and thus $n_i^{out} = n^{out} = n - n^{in} - n^*$) for all $i \in S$. Observe however that in-, out- and intra-degree cannot take any values (even when these add to n). There are two ways of counting the number of edges across two adjacent iterated neighborhoods: as the total number of edges going from N_d^X to N_{d+1}^X (thus using out-degrees) and as the total number of edges going from N_{d+1}^X to N_d^X (thus using in-degrees) for any $d \geqslant 0$. This is written

$$\sum_{j \in N_X^d} n_j^{out} = \sum_{k \in N_X^{d+1}} n_k^{in}. \tag{7}$$

In homogeneous networks the expression boils down to $n^{out} \cdot n_X^d = n^{in} \cdot n_X^{d+1}$, for all $d \geqslant 0$. This allows the size of N_X^d to be written as $n_X^d = x \left(n^{out}/n^{in}\right)^d$, for all $d \geqslant 0$, an expression which tends to zero if $n^{in} > n^{out}$ and thus contradicts our assumption of (S, g) being of infinite order. Thus we must have $n^{in} \leqslant n^{out}$ (if $n^{in} = n^{out}$, the growth of X_d is linear, when $n^{in} < n^{out}$ growth is geometric) and will only consider such networks in this section. The following proposition then directly obtains from Proposition 1.

Proposition 2. *Suppose $\beta \in (\beta_{\ell-1}, \beta_\ell]$, for $\ell \geqslant 1$. There is shell-wise contagion in the homogeneous network (S, g) if and only if intra-degree is large enough, $n^* \geqslant \ell$.*

In a homogeneous population, a necessary and sufficient condition for shell-wise contagion is that the number of intra-(iterated-)neighborhood interactions is large enough, and larger when the cooperation penalty is larger. As an example, consider a tree and begin with $X = \{r\} \cup N_r$, where r is a random player in the tree. The iterated neighborhoods of X are characterized by $n_i^* = 0$, $n_i^{in} = 1$ and $n_i^{out} = n - 1$, for all $i \in S - X$. Even when the payoff matrix is most favorable to cooperation ($\beta \in (0, (\tau - \pi)/n]$, i.e. cooperation outperforms defection when played by at least one extra C-player) shell-wise contagion is impossible. For any choice of the seed group X, iterated neighborhoods in a tree have no intra-degree and thus cooperation cannot diffuse. The infinite line (the non-periodic 1-dimensional lattice) is a particular tree, for which any initial connected X yields $n_i^{in} = n_i^{out} = 1$ and $n_i^* = 0$, for all $i \in S - X$. Shell-wise contagion is thus impossible on the line.[6]

A richer situation obtains when the homogeneity assumption is partly relaxed by imposing only one dimension of identity.

[6] In Eshel et al. (1998), where imitative learning is based on the average payoff earned by each strategy rather than on the largest payoff earned, there is contagion starting from an initial cluster of three C players. The learning rule used here is different, and thus the results are.

Proposition 3. *Suppose $\beta \in (\beta_{\ell-1}, \beta_\ell]$, for $\ell \geqslant 1$. There is shell-wise contagion:*
(i) in the out-degree homogeneous network (S,g) if and only if intra-degree is large enough: $n_i^* \geqslant \ell, \forall i \in S - X$;
(ii) in the intra-degree homogeneous network (S,g) only if intra-degree is large enough: $n^* \geqslant \ell$;
(iii) in the in-degree homogeneous network (S,g) if intra-degree is large enough: $n_i^* \geqslant \ell, \forall i \in S - X$.

Proof. (i) In the out-degree homogeneous network we have $n_i^{out} = n^{in}, \forall i \in S$. Condition 6 is rewritten as $n_k^{in} \leqslant n-\ell-n^{out}$, or equivalently $n_k^* \geqslant \ell, \forall k \in S-X$. (ii) In the intra-degree homogeneous network where $n_i^* = n^*, \forall i \in S$, Condition 6 becomes $\max_{k\in N_i^+ \cap N_X^{d_{X,i}}}\{n_k^{in}\} \leqslant \max_{j\in N_i^+ \cap N_X^{d_{X,i}-1}}\{n_j^{in}\} + n^* - \ell$, for all $i \in S-X$. Suppose $\ell > n^*$. By assumption, in any iterated neighborhood $n_j^{in} \leqslant n-2$. Thus for the players in N_X^1, if Condition 6 holds it implies that $n_k^{in} \leqslant n-2-(\ell - n^*)$, for all $k \in N_X^1$. Turn to the players in N_X^2. Condition 6 is written $\max_{k\in N_i^+ \cap N_X^2}\{n_k^{in}\} \leqslant \max_{j\in N_i^+ \cap N_X^1}\{n_j^{in}\} + n^* - \ell$, for all $i \in N_X^2$. But we have seen that $n_k^{in} \leqslant n-2-(\ell-n^*)$, for all $k \in N_X^1$. Thus Condition 6 implies that $n_k^{in} \leqslant n-2-2(\ell-n^*)$, for all $k \in N_X^2$. Repeating the argument will imply the existence of a finite d for which in-degree becomes negative, a contradiction which proves the result. (iii) Finally, in the in-degree homogeneous network where $n_i^{in} = n^{in}, \forall i \in S$, Condition 6 is written $\ell \leqslant \max_{j\in N_i^+ \cap N_X^{d_{X,i}-1}}\{n_j^*\}, \forall i \in S-X$, which is implied by the condition $n_i^* \geqslant \ell$ for all $i \in S-X$. $\square$

An immediate consequence of part (ii) of Proposition 3 is that the existence of (at least some amount of) intra-degree is necessary for shell-wise contagion. If all players have zero intra-degree for any finite X, there cannot be contagion. These results again emphasize the fundamental role played by intra-neighborhood interactions. To contrast intra-degree in iterated neighborhoods and clustering, a few examples are considered. Consider (S,g) such that the population is divided into regions of m players each. Each player in a region interacts with n^* players in the region he belongs to. The regions are arranged in a line and each player interacts with one (or more) player(s) in each neighboring region, so the whole graph looks like a bracelet, as displayed in the figures below, with $n=4$ and $n^*=2$ in Figure 2; $n=5$, $n^*=3$ and $n^{in}=1$ in Figure 3; and a partly homogeneous situation, with $n=5$, $n^*=2$ but $n_i^{in} \in \{1,2\}$, in Figure 4.

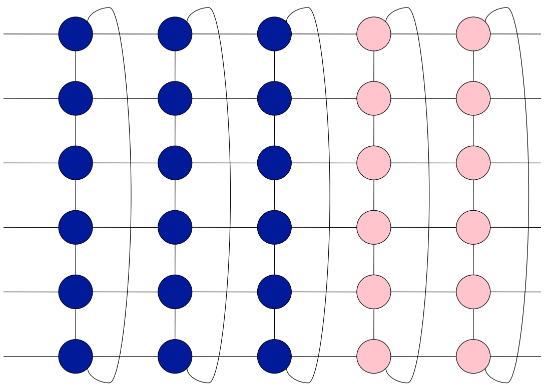

Figure 2: An illustration of contagion with $n^{in} = n^{out} = 1$ and $n^* = 2$.

Begin with Figure 2. This is the 2-dimensional periodic lattice with Newman neighborhood. Take as the starting set X three adjacent C-playing regions on the left. Then (S,g) is homogeneous as $n^{in} = n^{out} = 1$ and $n^* = 2$, for a total of $n=4$ neighbors for each player. X seeds shell-wise contagion for $\beta \in (0, (\tau-\pi)\cdot 2/4]$. This is a typical illustration of a situation in which interactions take place mostly between players with the same strategy. Starting with segregated regions, it is visible that Cs do well, as they have a majority of their interactions among themselves, and for the same reason Ds do poorly. Thus even with a large penalty to cooperation, cooperation can diffuse provided it starts in a well-protected region, and interaction is mostly taking place within iterated neighborhood (50% of the interactions of any agent in Figure 2 are within the iterated neighborhood to which the agent belongs). It is worth noting that there is no transitivity or clustering in the network (there are no triangles). So intra-degree, i.e. the existence of edges between players which belong to a given iterated neighborhood of X, and transitivity (or clustering) are different notions. The d-shells are $3/4 = 0.75$-cohesive (agents at the border have 3 of their interaction within the d-shell, other inside the shell have all their interactions within the d-shell.

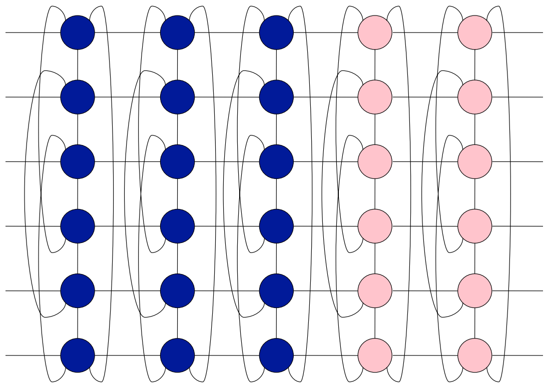

Figure 3: An illustration of contagion with $n = 5$ and $n^* = 3$.

Figure 3 exemplifies further the absence of a need for transitivity, with $n = 5$ and $n^* = 3$. Again no edge exists between any pair of neighbors of a given player (the network has no cycles of period less than 4) and yet X seeds shell-wise contagion for $\beta \in (0, (\tau - \pi) \cdot 3/5]$, a larger interval of acceptable values than in Figure 2. This time d-shells are $4/5 = 0.8$-cohesive (agents at the border have 4 of their interaction within the d-shell, other inside the shell have all their interactions within the d-shell).

Imagine now enlarging neighborhoods by connecting all the players in a given region (this is not represented). Then $n = 7$, with $n^* = 5$ and again $n^{in} = n^{out} = 1$. This time X seeds shell-wise contagion for $\beta \in (0, (\tau - \pi) \cdot 5/7]$. The acceptable interval has grown even larger. However, agents' neighborhoods are now highly clustered: among the $7 \cdot 6/2 = 21$ possible pairwise links which can form, $5 \cdot 4/2 = 10$ exist, i.e. $cl_i = 10/21 = 0.48$ for any $i \in S$. Iterated neighborhoods of X are also highly transitive: they form cliques and thus have a transitivity of 1. Finally in this case d-shells are $6/7 = 0.86$-cohesive. So local overlap is good for contagion, but unnecessary, which is in contrast with many findings in the literature (see for instance Abramson and Kuperman 2001; Vega-Redondo et al. 2005; Roca et al. 2009).

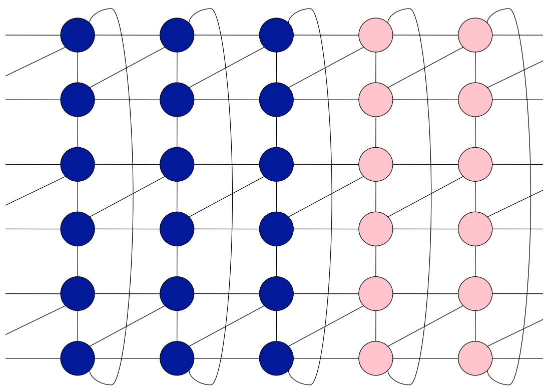

Figure 4: An illustration of contagion with $n = 5$ and $n^* = 3$.

As a final example, look at the partly homogeneous network in the Figure 4. Now $n = 5$, $n^* = 2$ but $n_i^{in} \in \{1, 2\}$. According to Proposition 3, whenever $\beta > (\tau - \pi) \cdot 2/5$ we should not expect contagion. We see in the figure that the threshold is also sufficient: when $\beta \in (0, (\tau - \pi) \cdot 2/5]$ Condition 6 holds, and so X seeds shell-wise contagion. We have clustering in agents' neighborhoods: among the $5 \cdot 4/2 = 10$ possible pairwise links, 3 exist, i.e. $cl_i = 3/10 = 0.3$ for any $i \in S$.

It is interesting to contrast Condition 6 with the notion of cohesion as used in Morris (2000). The cohesion of X_d is easily seen to be $\min_{i \in N_X^d} (n_i^{in} + n_i^*)/n$. Thus sufficiently cohesive shells will imply a large right hand side term in Condition 6, but cannot alone guarantee that the entire condition holds. Clustering and transitivity are harder to compute in general, but have already been seen necessary for contagion in the examples above.

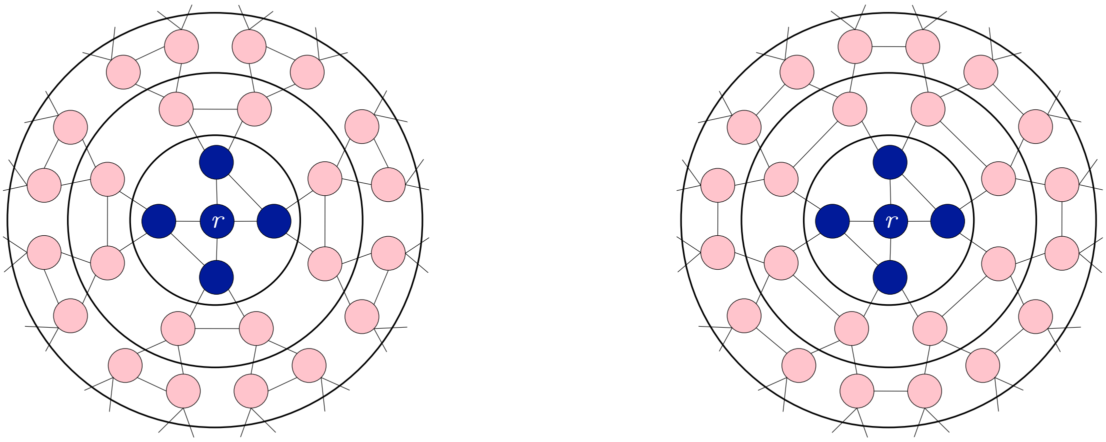

Figure 5: Contagion in a network with geometric neighborhood growth.

As a last illustration, Figure 5 displays two homogeneous network structures with geometric neighborhood growth in which contagion takes place. Starting from a seed group $X = \{r\} \cup N_r$ (one player and his 4 neighbors), both network are characterized by $n = 4$, $n^* = 1$ and $n_i^{in} = 1$. From Proposition 2 there is contagion if and only if $n^* \geqslant \ell$, and thus X seeds shell-wise contagion for $\beta \in (0, (\tau - \pi)/4]$. However the size of N_X^d is written $n_X^d = n_X^{d-1}\left(n^{out}/n^{in}\right)$, for all $d \geqslant 2$, so that $x_d = 5 + 8(1 + 2 + \ldots + 2^{d-1}) = 5 + 8(2^d - 1)$, i.e. we have very fast (geometric) iterated-shell growth in both networks. Interestingly, the network in the left panel exhibits some amount of transitivity (there are triangles) whereas the network in the right panel has no cycle of length less than 4 (of which it has many) except in the seed group. In terms of speed of contagion however both networks behave the same way.

3.2.2. Inhomogeneous networks

Contagion in inhomogeneous networks where in-, out- and intra-degree can be anything is much harder to characterize. However the importance of intra-shell interaction is maintained, as summarized in the following proposition which establishes both a necessary and a sufficient condition for contagion.

Proposition 4. *Suppose $\beta \in (\beta_{\ell-1}, \beta_\ell]$, for $\ell \geqslant 1$.*
(i) A connected finite-sized set $X \subset S$ does not seed shell-wise contagion in (S, g) if $n_i^* < \ell$ for all $i \in S - X$;
(ii) There is shell-wise contagion in (S, g) if there exists a connected finite-sized set $X \subset S$ such that $n_i^* \geqslant (n+\ell)/2 - 1$, for all $i \in S - X$.

Proof. The first part is similar to point (ii) in Proposition 3. If $n_i^* \leqslant \ell - 1$, Condition 6 implies that $\max_{k \in N_i^+ \cap N_X^1}\{n_k^{in}\} + \ell \leqslant \max_{j \in N_i^+ \cap X}\{n_j^{in} + n_j^*\} \leqslant n - 2 + \ell - 1, \forall i \in N_X^1$. So $n_k^{in} \leqslant n - 3, \forall k \in N_X^1$. Turn to the players in N_X^2. Condition 6 yields $\max_{k \in N_i^+ \cap N_X^2}\{n_k^{in}\} + \ell \leqslant \max_{j \in N_i^+ \cap N_X^1}\{n_j^{in} + n_j^*\} \leqslant n - 3 + \ell - 1$. So $n_k^{in} \leqslant n - 4$, for all $k \in N_X^2$. Repeating the argument will imply the existence of a finite d for which in-degree becomes negative, a contradiction which proves the result.

For the second part, assume $n_i^* \geqslant n^*$ for all $i \in S - X$. Then $1 \leqslant n_i^{in} \leqslant n - 1 - n^*$ and Condition 6 holds if the largest possible value of the left-hand side expression is smaller than the smallest possible value of the right-hand side expression, i.e. $n - 1 - n^* \leqslant n^* + 1 - \ell$, that is to say $n^* \geqslant (n + \ell)/2 - 1$. □

Point (i) in Proposition 4 generalizes point (ii) in Proposition 3 to a heterogeneous network. Point (ii) in Proposition 4 formulates a sufficient condition in heterogeneous networks, but a fairly weak one: the cutoff value of n_i^* is much larger than ℓ. Between ℓ and the halved-sum of n and ℓ, it is difficult to characterize what happens. We nonetheless can conclude that systematic low values of intra-degree prevent contagion, whereas systematic high values of intra-degree entail contagion.

Typical examples of inhomogeneous networks are the lattices very often studied in the literature. Figure 6 below displays a two-dimensional lattice with Moore neighborhood structure (each agent has 8 neighbors, the 4 obvious ones on the lattice with Newman neighborhood, and the additional four players "at the corners"). The $\mathbf{n}^*, n^{in}$-pairs are provided for the iterated neighborhoods of an initial seed set $X = \{r\} \cup N_r$.

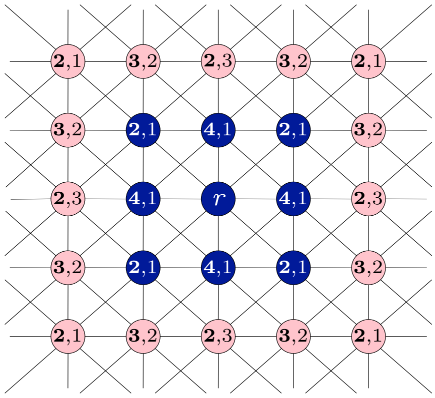

Figure 6: Two-dimensional lattice with Moore neighborhood.

Suppose $X = \{r\} \cup N_r$. When $\ell = 1$, we are unable to use Proposition 4 to characterize contagion, but we can directly use Proposition 1, as for any $i \in N_X$, $\max_{k \in N_i^+ \cap N_X} \{n_k^{in}\} + \ell \leqslant \max_{j \in N_i^+ \cap N_X} \{n_j^{in} + n_j^*\}$. Indeed if $\max_{k \in N_i^+ \cap N_X} \{n_k^{in}\} = 2$, then $\max_{j \in N_i^+ \cap N_X} \{n_j^{in} + n_j^*\} = 2 + 1 = 3$, whereas if $\max_{k \in N_i^+ \cap N_X} \{n_k^{in}\} = 3$, then $\max_{j \in N_i^+ \cap N_X} \{n_j^{in} + n_j^*\} = 4 + 1 = 5$: the inequality holds for $\ell = 1$ in both cases. When $\ell = 2$ Condition 6 does not hold anymore, i.e. there is not shell-wise contagion (it can be showed there is still contagion, but it is not shell-wise).

4. Contagion dynamics in Coordination Games

Players engage in the symmetric two-by-two Coordination Game with positive payoffs given in Table 4, with $\pi > \tau$ and $\rho > \tau$. We assume $\pi > \sigma$, which implies that (C, C) is the Pareto-dominant equilibrium or equivalently that C is the efficient action, and $\tau < \pi$, to rule out the stag-hunt game. In addition, we assume $\rho + \sigma < \tau + \pi$, implying (D, D) risk dominates (C, C).

	C	D
C	ρ	σ
D	τ	π

Table 2: Payoffs to the row player in the Coordination Game.

The aggregate payoff to player i is written

$$P_i = \sum_{j \in N_i} p(\omega_i, \omega_j) = \begin{cases} v_i(\rho - \sigma) + n\sigma & \text{if } \omega_i = C, \\ v_i(\tau - \pi) + n\pi & \text{if } \omega_i = D. \end{cases}$$

Given configuration ω, the imitative reaction for player i to group Z is uniquely defined as

$$r(Z, i) = C \text{ if } P_{d(i)} \leqslant P_{c(i)} \tag{8}$$

and $r(Z, i) = D$ otherwise, with $d(i)$ and $c(i)$ defined as in Section 2.2. The Coordination Game, unlike the Prisoner's Dilemma, is characterized by the fact that the payoff to each strategy is increasing in the number of players endowed with that strategy in the augmented neighborhood of the focal player. This results in a proposition very similar to Proposition 1.

Proposition 5. *There is shell-wise contagion in (S, g) if and only if there exists a connected finite-sized set $X \subset S$ such that*

$$\min_{j \in N_i^+ \cap N_X^{d_{X,i}-1}} \{n_j^{out}\} \leqslant n \frac{\rho - \pi}{\rho - \sigma} \tag{9}$$

for all $i \in S - X$.

Proof. Given the initial set X, if contagion has reached the d-neighborhood of X then any $i \in N_X^{d+1}$ playing D switches to C if and only if Condition 9 holds. Now $i \in N_X^d$ playing C sees a player in N_X^{d-1} who interacts only with Cs, collecting payoff $n\rho$, the largest possible payoff, and thus never switches behavior. Applying the reasoning iteratively implies that $X_{d+1} = R(X_d)$ for all $d \geqslant 0$, and thus X seeds shell-wise contagion if and only if Condition 9 holds for all $i \in S - X$. □

The situation is different from the one studied in the Prisoner's Dilemma: here, largest payoffs for both actions are collected inside the C- and D-clusters, rather than at the border between the clusters. The constraint is explicitly on neighborhood growth: for efficient play to prevail there must be agents whose neighborhoods grow slowly, i.e. who have small out-degree. Imposing a low out-degree implies interactions which are mostly within the C-cluster for C players, i.e. we are interested in shells displaying large amount of cohesion (or, to be more precise, such that enough players' neighborhoods display large amounts of cohesiveness). The simplest characterization obtains in out-degree homogeneous networks, where there is shell-wise contagion if and only if out-degree is small enough, $n^{out}/n \leqslant (\rho - \pi)/(\rho - \sigma)$. Formulated in terms of cohesion, this means that d-shells should be at least $1 - (\rho - \pi)/(\rho - \sigma)$-cohesive (see the discussions on the implications of cohesion in Young 1998 and Morris 2000). Note that when D is risk-dominant the ratio $(\rho - \pi)/(\rho - \sigma)$ is strictly less than 1/2, i.e. the growth rate of iterated neighbourhoods cannot be too large. Also note that, as the network is regular, we could formulate an obvious sufficient condition on intra-degree such that any agent's out-degree is sufficiently small. Finding a necessary condition on intra-degree is however much more difficult.

5. Discerning interaction from information

Recent literature has explored the consequences of distinguishing interaction and information neighborhoods (see Alós-Ferrer and Weidenholzer 2008; Mengel 2009). In such a context, players interact with their neighbors, but exchange payoff information beyond their neighborhoods. Eshel et al. (1999) already have this feature in their model, but with an additional assumption of conservative behavior: though information is acquired beyond the interaction neighborhood, adopting the new behavior is only possible if it is played in the interaction neighborhood. This distinction is particularly relevant in the Prisoner's Dilemma, as payoffs to the D-players are larger at the border than inside D-clusters. Thus without the conservative assumption, one could imagine D-players switching to C within a cluster, while those D-players at the border would not, the resulting diffusion process being then much harder to study.

Alós-Ferrer and Weidenholzer (2008) show that when information can be obtained beyond players' interaction neighborhoods there is efficient contagion in the Coordination Game. The intuition goes as follows. Letting $\delta \geqslant 2$ be the information distance, the interactions of player i are with players in N_i as before, while information is obtained from players in $\{i\}_\delta = \{i\} \cup N_i \cup \ldots \cup N_i^\delta$ (i.e. from the δ-shell of i). Consider a finite group X of C-players such that there is at least one C-player interacting only with C-players (and thus collecting the largest payoff $n\rho$) in any player's information neighborhood. Call such a player a top-scorer. All C-players at the border of X (i.e. having both strategies in their neighborhood) have a top-scorer at distance 1. Obviously players in X do not switch to D. Furthermore, all D-players in N_X have a top-scorer at distance 2, and more generally all D-players in $X_{\delta-1}$ have a top-scorer within distance δ. After one period contagion will thus extend to $X_{\delta-1}$, and the reasoning can be iterated.

With the conservative assumption, the same result of efficient contagion obtains in the Coordination Game, iterated-neighborhood converting one after another as time passes (all D-players in N_{X_d} have a top-scorer at distance 2 in X_d, and $\delta \geqslant 2$).

The Prisoner's Dilemma offers a different perspective, and we make the conservative assumption to avoid complicated, discontinuous types of contagion. Let the information distance be $\delta \geqslant 2$. The results are summarized in the proposition below.

Proposition 6. *Suppose $\beta \in (\beta_{\ell-1}, \beta_\ell]$, for $\ell \geqslant 1$. There is shell-wise contagion in (S, g) if and only if there exists a finite-sized set $X \subset S$ such that*

$$n_i^{in} + \ell \leqslant n \tag{10}$$

for all $i \in S - X$.

Proof. Given the initial set X, assume contagion has reached the d-neighborhood of X ($\omega_i = C$ for all $i \in X_d$) where d is a non-negative integer number. When $\delta \geqslant 2$, the C-players in N_X^d and the D-players in N_X^{d+1} are in exactly the same situation. Each of them sees at least one player in N_X^{d-1} who only play with Cs. In addition, each of them sees the players in the intersection of his augmented neighborhood with N_X^{d+1}. If all the D-players in N_X^{d+1} play with (strictly) less than $n - \ell$ Cs, the D-players convert to C. If there is (at least) one D-player in N_X^{d+1} who plays with more than $n - \ell$ Cs, him and possibly others who see him will not switch. The same is true for the C-players in N_X^d, under exactly the same condition: if there is (at least) one D-player in N_X^{d+1} who plays with more than $n - \ell$ Cs, those Cs who see him will switch to D. As a result all, $R(X_d) = X_{d+1}$, if and only if $n_i^{in} + \ell \leqslant n$ for all $i \in S - X$. □

This very simple condition contrasts with the one obtained when the interaction and information neighborhoods coincide. In the former situation, interacting with players in the same situation (i.e. facing the same choice) was essential for shell-wise contagion to take place. In the present context, when information can be acquired beyond the interaction neighborhood, the extent to which players facing the same situation interact is no longer relevant. It is interesting to note that we have here a fast neighborhood growth condition, rather than a slow neighborhood growth condition. So things are to a large extent reverted, compared to the case of identical interaction and information neighborhoods. To see this, observe that even in a tree it is now possible to get efficient contagion. Starting with $X = \{r\} \cup N_r$, where r is a random player in the tree, and constructing the iterated neighborhoods of X yields $n_i^* = 0$, $n_i^{in} = 1$ and $n_i^{out} = n - 1$, for all $i \in S - X$. Condition 10 is written $\ell \leqslant n - 1$, i.e. efficient contagion obtains when $\beta \in (0, (\tau - \pi) \cdot (n-1)/n]$. Even the least favorable payoff conditions (when cooperation needs to be played by $n-1$ additional players to outperform defection) allow efficient shell-wise contagion to happen.

6. Conclusion

This paper has examined the conditions under which efficient behavior diffuses in a network, starting from an initial seed group. Our focus is on imitative learning, i.e. we assume that players imitate the best observable action in their neighborhood. For arbitrary regular networks, we find that efficient contagion in a Prisoner's Dilemma depends strongly on the extent to which play mostly happens within (rather than across) the iterated neighborhoods of the initial seed group. These interactions take place between players who are confronted with the same decision at the same time, i.e. who are at the same distance from the initial seed group. Interactions with players who were confronted with the decision yesterday or who will be confronted with it tomorrow turn out to be less crucial. Depending on how homogeneous the network is, the number of intra-(iterated-)neighbourhood interactions can act as a sufficient or a necessary condition for contagion by efficient play. In all cases these connections play a central part. The larger the premium for defection is in the game, the more it is important to have these connections in place. When the premium to defection becomes too large, cooperative behaviour cannot be sustained. We show that slow neighborhood growth is not necessary for efficient contagion, and contrast our notion of intra-(iterated-)neighbourhood interactions with clustering, transitivity and cohesion. We show that transitivity (or clustering) is not required for efficient contagion. Cohesion is more closely related to our structural property, though the two notions remain distinct, as we illustrate in several examples. The main result of the paper is thus that contagion can be obtained, and sometimes demands a particular type of local structure, one which is not as intuitive as friendship commonality (the property that friends of my friends should be my friends as well).

In the Coordination Game, the growth of the efficient cluster requires that the cluster has fewer outward than inward connections. When the efficient and risk-dominant actions differ, there should always be strictly less than $1/2$ of outward connections. The condition is similar to what other papers have identified earlier in the literature (see Morris 2000). As risk dominance gets stronger, efficient contagion requires slower neighborhood growth.

We then examine the situation in which information neighborhoods extend beyond interaction neighborhoods. In that case efficient contagion obtains in the Coordination Game, as identified elsewhere in the literature (Alós-Ferrer and Weidenholzer 2008). For the Prisoner's Dilemma, adding an assumption of conservative learning yields an upper bound on in-degree. In-degree should be low for efficient contagion to happen, a configuration in which typically neighborhood growth could be very fast. Therefore, extended information neighborhoods are not only favorable to cooperation, but actually permit the most rapid diffusion of efficient behavior on regular networks, as contagion takes place in networks in which neighborhoods grow fast.

[1] G. Abramson and M. Kuperman, *Social games in a social network*, Physical Review E **63** (2001), no. 3, 30901.
[2] C. Alós-Ferrer and S. Weidenholzer, *Contagion and efficiency*, Journal of Economic Theory **143** (2008), no. 1, 251–274.
[3] L. Anderlini and A. Ianni, *The dynamics of norms*, ch. Learning on a torus., pp. 87–107, Cambridge University Press, 1997.
[4] J. Apesteguia, S. Huck, and J. Oechssler, *Imitation–theory and experimental evidence*, Journal of Economic Theory **136** (2007), no. 1, 217–235.
[5] J. Bergin and D. Bernhardt, *Cooperation through imitation*, Games and Economic Behavior **67** (2009), no. 2, 376–388.
[6] J. Bergin and B.L. Lipman, *Evolution with state-dependent mutations*, Econometrica **64** (1996), no. 4, 943–956.
[7] S.K. Berninghaus and U. Schwalbe, *Evolution, interaction, and nash equilibria*, Journal of Economic Behavior and Organization **29** (1996), no. 1, 57–85.
[8] K. Binmore and L. Samuelson, *Muddling through, noisy equilibrium selection*, Journal of Economic Theory **74** (1997), 235–265.
[9] L.E. Blume, *The statistical mechanics of strategic interaction*, Games and Economic Behavior **5** (1993), 387–424.
[10] L.E. Blume, *The statistical mechanics of best-response strategy revision*, Games and Economic Behavior **11** (1995), no. 2, 111–145.
[11] E.J. Cartwright, *Contagion and the speed of adjustment in small worlds.*, International Game Theory Review **9** (2007), no. 4, 689–704.
[12] P. Duersch, A. Kolb, J. Oechssler, and B.C. Schipper, *Rage against the machines: how subjects play against learning algorithms.*, Economic Theory **43** (2010), no. 3, 407–430.
[13] J. Durieu, H. Haller, and P. Solal, *Cognitive economics: New trends*, ch. Contagion and Dominating Sets, pp. 135–152, Emerald Group Pub Ltd, 2007.
[14] G. Ellison, *Learning, local interaction and cooperation*, Econometrica **61** (1993), 1047–1071.
[15] G. Ellison, *Basins of attraction, long-run stochastic stability, and the speed of step-by-step evolution*, Review of Economic Studies **67** (2000), no. 1, 17–45.

[16] I. Eshel, L. Samuelson, and A. Shaked, *Altruists, egoists, and hooligans in a local interaction model*, American Economic Review **88** (1998), 157–179.
[17] I. Eshel, E. Sansone, and A. Shaked, *The emergence of kinship behavior in structured populations of unrelated individuals*, International Journal of Game Theory **28** (1999), no. 4, 447–463.
[18] B.A. Huberman and N.S. Glance, *Evolutionary games and computer simulations*, Proceedings of the National Academy of Sciences **90** (1993), no. 16, 7716–7718.
[19] T. Jun and R. Sethi, *Neighborhood structure and the evolution of cooperation*, Journal of Evolutionary Economics **17** (2007), no. 5, 623–646.
[20] M. Kandori, G.J. Mailath, and R. Rob, *Learning, mutation, and long run equilibria in games*, Econometrica **61** (1993), 29–56.
[21] M.B. Lieberman and S. Asaba, *Why do firms imitate each other?*, Academy of Management Review **31** (2006), no. 2, 366–385.
[22] D. López-Pintado, *Contagion and coordination in random networks*, International Journal of Game Theory **34** (2006), no. 3, 371–381.
[23] F. Mengel, *Conformism and cooperation in a local interaction model*, Journal of Evolutionary Economics **19** (2009), no. 3, 397–415.
[24] S. Morris, *Contagion*, Review of Economic Studies (2000), 57–78.
[25] M.A. Nowak, S. Bonhoeffer, and R.M. May, *Spatial games and the maintenance of cooperation*, Proceedings of the National Academy of Sciences **91** (1994), no. 11, 4877–4881.
[26] M.A. Nowak and R.M. May, *Evolutionary games and spatial chaos*, Nature **359** (1992), no. 6398, 826–829.
[27] M.A. Nowak and R.M. May, *The spatial dilemmas of evolution*, International Journal of Bifurcation and Chaos **3** (1993), 35–35.
[28] H. Ohtsuki, C. Hauert, E. Lieberman, and M.A. Nowak, *A simple rule for the evolution of cooperation on graphs and social networks*, Nature **441** (2006), 502–505.
[29] A.V. Outkin, *Cooperation and local interactions in the prisoners' dilemma game*, Journal of Economic Behavior and Organization **52** (2003), no. 4, 481–503.
[30] M. Pingle and R.H. Day, *Modes of economizing behavior: Experimental evidence*, Journal of Economic Behavior and Organization **29** (1996), no. 2, 191–209.
[31] A.J. Robson and F. Vega-Redondo, *Efficient equilibrium selection in evolutionary games with random matching*, Journal of Economic Theory **70** (1996), no. 1, 65–92.
[32] C.P. Roca, J.A. Cuesta, and A. Sánchez, *Effect of spatial structure on the evolution of cooperation*, Physical Review E **80** (2009), no. 4, 46106.
[33] L. Samuelson, *Evolutionary games and equilibrium selection*, Mit Press, 1997.
[34] K.H. Schlag, *Why imitate, and if so, how? a bounded rationality approach to multi-armed bandits*, Journal of Economic Theory **78** (1998), 130–156.
[35] H.A. Simon, *A behavioral model of rational choice*, The Quarterly Journal of Economics **69** (1955), no. 1, 99–118.
[36] F. Vega-Redondo, M. Marsili, and F. Slanina, *Clustering, cooperation, and search in social networks.*, Journal of the European Economic Association **3** (2005), no. 2/3, 628–638.
[37] H.P. Young, *The evolution of conventions*, Econometrica **61** (1993), 57–84.
[38] H.P. Young, *Individual strategy and social structure: An evolutionary theory of institutions*, Princeton University Press, 1998.